1. Sturm sequences

1. Let $f(x) \in \mathbb{C}[x]$ be a nonzero polynomial.

(a) Let $\alpha \in \mathbb{C}$ be a root of f. Then α is a root of multiplicity r of f if and only if it is a root of multiplicity $r-1$ of the polynomial f'.

(b) In the previous claim, is the assumption about α being a root of f necessary?

2. The equation $f(x) = 0$ ($f \in \mathbb{C}[x]$, $f \neq 0$) has exactly $\deg f$ complex solutions, if each is counted with its multiplicity. Using the previous exercise design an algorithm for finding the number of pairwise different complex roots of f.

3. Use Sturm sequence to find out the number of real roots of the polynomial

$$f(x) = 2x^3 - 6x^2 + 1,$$

and separate them, i.e. find the disjoint bounded intervals in $\mathbb{R}$ such that each of them contains exactly one of the roots.