8. Zariski topology.

48. Let X be an irreducible topological space. Prove that each nonempty open set in X is dense.

49. Let $S \subset \mathbb{A}^n$ be an arbitrary set. Prove that $V(I(S)) = \overline{S}$, where $\overline{S}$ denotes the closure of the set S with respect to Zariski topology on $\mathbb{A}^n$. In other words you should prove that $V(I(S))$ is the smallest algebraic variety containing S. (So far we just briefly mentioned it, without any proof.)

50. Let $I = (xy, yz, xz) \subset \mathbb{C}[x, y, z]$. Decompose $V(I)$ into irreducible components.

51. Show that the ideal $(xy, yz, xz) \subset \mathbb{C}[x, y, z]$ cannot be generated by less than three polynomials.

52. Let $I = (x^2 + y^2 + z^2, x^2 - y^2 - z^2 + 1) \subset \mathbb{C}[x, y, z]$. Decompose $V(I)$ into irreducible components.

53. Let $I = (x - yz, xz - y^2) \subset \mathbb{C}[x, y, z]$. Decompose $V(I)$ into irreducible components.

When decomposing a variety into irreducibles, you don't need to prove that the components are indeed irreducible. So far you don't have strong enough tools to do it.