

## 11. GRÖBNER BASIS.

- 66.** Prove that the graded reverse lexicographic ordering is indeed a monomial ordering.
- 67.** Prove that in the ring  $k[x]$  ( $k$  being a field), there is only one monomial ordering.
- 68.** Let  $>$  be a monomial ordering. For two different monomials  $x^\alpha, x^\beta$ , prove that if  $x^\alpha$  is divisible by  $x^\beta$ , then  $x^\alpha > x^\beta$ . Does also the reverse implication hold?
- 69.** Let the ring  $k[x_1, \dots, x_n]$  have a chosen monomial ordering. Let  $G = \{f_1, \dots, f_k\}$  be a Gröbner basis of an ideal  $I \subset k[x_1, \dots, x_n]$ . If  $\text{LT}(f_i) \mid \text{LT}(f_j)$  ( $i \neq j$ ), then also  $\{f_1, \dots, f_{j-1}, f_{j+1}, \dots, f_k\}$  is a Gröbner basis of the ideal  $I$ . Prove it.