NURBS solids of revolution

Martin Samuelcik
Department of Applied Informatics
Faculty of Mathemathics, Physics and Informatics
Comenius University, Bratislava, Slovakia
samuelcik@fmph.uniba.sk

Abstract

In our short paper, we describe basic types of solids of revolution in NURBS form. These solids will be represented as trivariate volumes. As input object for generation of solids, we use NURBS curves and surfaces. In the intermediate steps, we use representation of circle and full disc as NURBS curve and surface. We also present results in the form of visualization of created volumes.

Keywords: geometric modeling, NURBS volumes, solids of revolution

1 Introduction

The solids of revolution are an important part of geometric modeling. In this paper, we focus on one part of these solids, created by the rotation of curves or surfaces around some axis. It is one of the most useful modeling tools used also in commercial packages. In many cases such solids are represented only by their boundary. However it is often important to describe also interior of created solids.

The NURBS volumes are one possible choice of the representation for mathematical description of solid's interior. In this representation, we deal with several input parameters (control points, weights, degrees and knot vectors) that define more complex model [Samuelcik 2006]. These trivariate volumes are extension of similar approach for curves and surfaces [Piegl et al. 1995] and also extension of Bézier volumes [Lasser 1985]. To create NURBS volume as solid of revolution, we have to find only exact values of these parameters. Also we will create such solids from curves and surfaces in NURBS representation.

2 NURBS Volumes

NURBS volumes are a natural extension of NURBS curves and surfaces used in geometric modeling. Parameters that define NURBS volume are similar to the surface case, but there are new parameters due to new added parameter (direction).

NURBS volume is defined with

- three degrees *du, dv, dw*,
- three non-decreasing knot vectors $(u_0, u_1, \dots u_{mu})$, $(v_0, v_1, \dots, v_{mv})$, $(w_0, w_1, \dots, w_{mw})$,
- three-dimensional net of control points $V_{i,j,k}$ in E^3; $0<=i<=n_u$; $0<=j<=n_v$; $0<=k<=n_w$;
- for each control point $V_{i,j,k}$ real number (weight) $p_{i,j,k}$
- domain $<u_{du},u_{nu}+1)$x$< v_{dv},v_{nv}+1)$x$< w_{dw},w_{nw}+1)$
- $m_u=n_u+d_u+1$, $m_v=n_v+d_v+1$, $m_w=n_w+d_w+1$

Then NURBS volume is given analytically as

$$S(u,v,w) = \frac{\sum_{i=0}^{n_u}\sum_{j=0}^{n_v}\sum_{k=0}^{n_w} p_{i,j,k} V_{i,j,k} N_i^{d_u}(u) N_j^{d_v}(v) N_k^{d_w}(w)}{\sum_{i=0}^{n_u}\sum_{j=0}^{n_v}\sum_{k=0}^{n_w} p_{i,j,k} N_i^{d_u}(u) N_j^{d_v}(v) N_k^{d_w}(w)}$$

, where $N_i^{d_u}(u), N_j^{d_v}(v), N_k^{d_w}(w)$ are B-spline blending functions defined on particular knot vectors with given degrees. Parameters *u,v,w* are from domain, $u \in <u_{d_u}, u_{n_u+1}), v \in <v_{d_v}, v_{n_v+1}), w \in <w_{d_w}, w_{n_w+1})$. Figure 1 gives an example of NURBS volume with randomly generated control points.

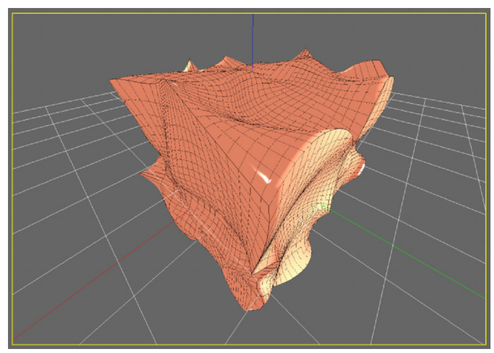

Figure 1. Random NURBS Volume.

3 Solids of Revolution

The solids of revolution in the basic form are solids that are created by rotating curve or surface around a given axis. If we are rotating the curve, we also rotate the whole space between curve and axis. In this case, we will focus on rotating by full circle angle (360 degrees). Solid of revolution can be created from two types of input objects, so we describe two types of solids of revolution. One type is created from a given curve by rotating space between that curve and the rotation axis. Rotating given surface around the given axis creates the second type.

To achieve our goal, we have to describe circle and full disc as NURBS curve and surface. There are many ways how to do it, we choose widely used square configurations. Figure 2 shows configuration for NURBS full circle, Figure 3 for NURBS full disc.

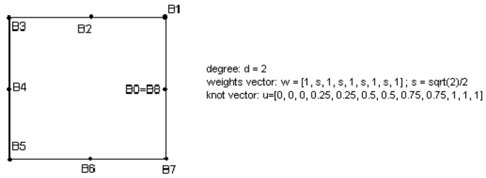

Figure 2. NURBS circle parameters.

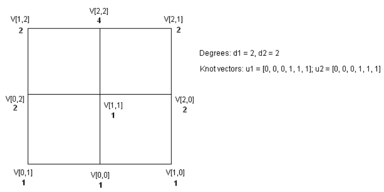

Figure 3. NURBS full disc parameters.

In our case, the axis of rotation will be the *z* coordinate axis; the rotation around arbitrary axis can be achieved using affine transformations of resulting volume. Based on this statement, succeeding NURBS circle and NURBS full disc are contained in *xy* coordinate plane and their center is in origin of coordinate frame.
Based on several described ways of creating basic solids of revolution, we distinguish the following types:

a) <u>NURBS solid of revolution from NURBS curve</u>
The first way to create NURBS solid of revolution from the NURBS curve is to simply put control points of NURBS full disc through each control point of given NURBS curve. If NURBS curve has control points V_0, $V_1,\ldots, V_n$ and weights $w_0, w_1,\ldots, w_n$, then NURBS volume has control points $P_{i,j,k} = V_i + W_{j,k} - W_{0,0}$ and weights $p_{i,j,k} = w_i * q_{j,k}$, where $W_{j,k}$ and $q_{j,k}$ are control points and weights of the NURBS full disc. The knot vectors and degrees of this NURBS volume are union of the knot vectors and degrees of given NURBS curve and the NURBS full disc.

b) <u>NURBS solid of revolution from NURBS surface</u>
In this situation, we rotate given surface around axis, this is achieved in NURBS form by putting the control points of NURBS circle in each of the control points of given NURBS surface. If given NURBS surface has control points $V_{i,j}$ and weights $w_{i,j}$, then NURBS volume has control points $P_{i,j,k} = V_{i,j} + W_k - W_0$ and weights $p_{i,j,k} = w_{i,j} * q_k$, where W_k and q_k are control points and weights of NURBS circle. Again the knot vectors and degrees of this NURBS volume are union of knot vectors and degrees of given NURBS surface and NURBS circle.

c) <u>NURBS solid of revolution from NURBS curve using NURBS surface</u>
First we create NURBS surface that represents space between given curve and given axis. The control points of such surface are control points of the given curve and points, which are projection of curve control points on given axis. Also degree, weights and knot vector for surface are copied from given curve, the second degree is 1 and the second knot vector is (0,0,1,1). When the surface is created, we can use case b) to make resulting NURBS solid of revolution.

4 Results and Future Work

We implemented described algorithms for creation of NURBS solids of revolution from NURBS curves and surfaces. It was implemented into our own system for modeling and visualization called GeomForge [Samuelcik 2006]. Figures 4 and 5 show NURBS solids created with our technique and visualized in our system.

In this paper, we described just the basic way to created solids of revolution. So our next goal will be description of more general solids in NURBS form. This generalization includes arbitrary angle and then arbitrary path of rotation.

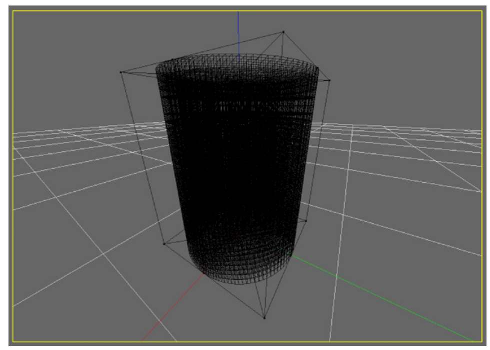

Figure 4. Cylinder as NURBS solid of revolution generated from NURBS curve.

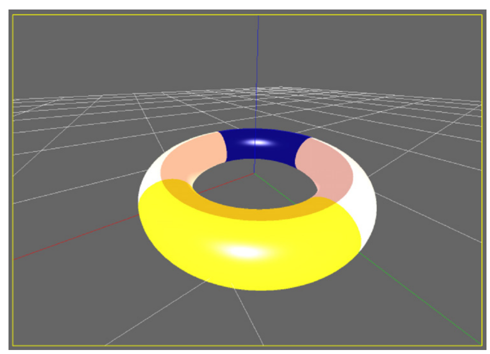

Figure 5. Torus as NURBS solid of revolution generated from NURBS surface.

5 Acknowledgments

This work has been supported by the VEGA grant Tools for Geometric Modeling No. 1/3024/06.

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