Modeling with Rational Bézier Solids

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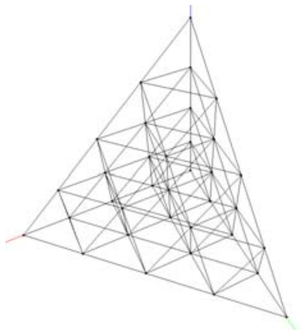
Rational Bézier tertahedra

Degree: *n*
Domain: tetrahedra *ABCD*
Control vertices: $V_{\mathbf{i}} \in E^3; w_{\mathbf{i}} \in R$

$\mathbf{i} = (i, j, k, l); |\mathbf{i}| = i + j + k + l = n; i, j, k, l \geq 0$



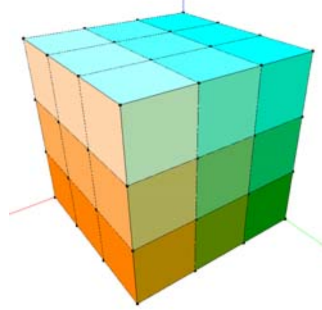
Definition: point U from domain, $\mathbf{u}=(u,v,w,t)$, $u+v+w+t=1$, $U=uA+vB+wC+tD$

$$RB^n(\mathbf{u}) = \frac{\sum_{|\mathbf{i}|=n} w_{\mathbf{i}} V_{\mathbf{i}} B^n_{\mathbf{i}}(\mathbf{u})}{\sum_{|\mathbf{i}|=n} w_{\mathbf{i}} B^n_{\mathbf{i}}(\mathbf{u})}; B^n_{\mathbf{i}}(\mathbf{u}) = \frac{n!}{i!\,j!\,k!\,l!} u^i v^j w^k t^l$$

Rational Bézier tensor solid

Degrees: *n, m, o*
Domain: box *ABCDEFGH*
Control vertices: $V_{\mathbf{i}} \in E^3; w_{\mathbf{i}} \in R$

$\mathbf{i} = (i, j, k); n \geq i \geq 0; m \geq j \geq 0; o \geq k \geq 0$



Definition: point U from domain, $\mathbf{u}=(u,v,w)$
$0 \leq u,v,w \leq 1$

$$RB^{n,m,o}(\mathbf{u}) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{o} w_{(i,j,k)} V_{(i,j,k)} B^n_i(u) B^m_j(v) B^o_k(w)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{o} w_{(i,j,k)} B^n_i(u) B^m_j(v) B^o_k(w)}$$

$$B^n_i(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

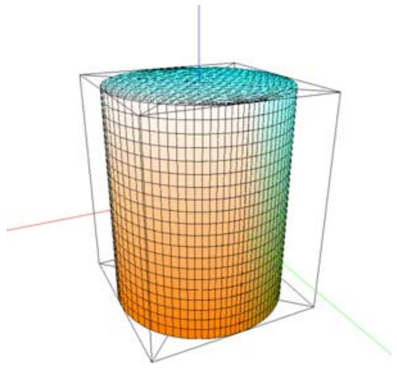
Transitional solids

Given:

- rational Bézier patch with given control points $P_{(i,j)}$ and weights $s_{(i,j)}$, where $i=0,..,n; j=0,\dots,m$
- rational Bézier curve with control points Q_i and weights r_i, where $i=0,\dots,o$

Rational Bézier tensor solid:

$$V_{(i,j,k)} = P_{(i,j)} + Q_k - Q_0; w_{(i,j,k)} = s_{(i,j)} * r_k$$
$$0 \leq i \leq n; \quad 0 \leq j \leq m; \quad 0 \leq k \leq o$$



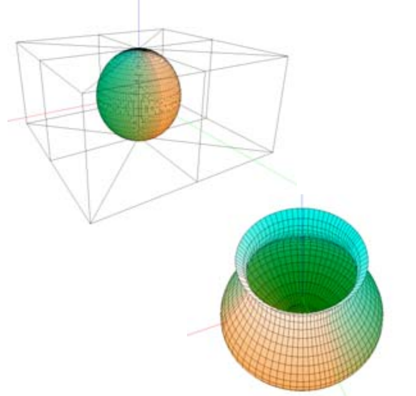
Rotational solids

Given:

- rational Bézier curve with control points Q_i and weights r_i, $i=0,\dots,o$

Rational Bézier tensor solid:

$$V_{(0,0,i)} = [Qx_i, Qy_i, Qz_i]; w_{(0,0,i)} = r_i;$$
$$V_{(0,1,i)} = [Qx_i + Qy_i, Qy_i - Qx_i, Qz_i]; w_{(0,1,i)} = r_i;$$
$$V_{(1,0,i)} = [Qx_i - Qy_i, Qx_i + Qy_i, Qz_i]; w_{(1,0,i)} = r_i;$$
$$V_{(0,2,i)} = [Qy_i, -Qx_i, Qz_i]; w_{(0,2,i)} = 2r_i;$$
$$V_{(1,1,i)} = [0, 0, Qz_i]; w_{(1,1,i)} = r_i;$$
$$V_{(2,0,i)} = [-Qy_i, Qx_i, Qz_i]; w_{(2,0,i)} = 2r_i;$$
$$V_{(1,2,i)} = [Qy_i - Qx_i, -Qx_i - Qy_i, Qz_i]; w_{(1,2,i)} = 2r_i;$$
$$V_{(2,1,i)} = [-Qx_i - Qy_i, Qx_i - Qy_i, Qz_i]; w_{(2,1,i)} = 2r_i;$$
$$V_{(2,2,i)} = [-Qx_i, -Qy_i, Qz_i]; w_{(2,2,i)} = 4r_i;$$
$$i = 0,\dots,o$$



Twisted solids

Given:

- rational Bézier patch with given control points $P_{(i,j)}$ and weights $s_{(i,j)}$, where $i=0,..,n; j=0,\dots,m$
- rational Bézier curve with control points Q_i and weights r_i, where $i=0,\dots,o$
- angle α

Rational Bézier r tensor solid:

$$V_{(i,j,k)} = (P_{(i,j)} + Q_k - Q_0) * \begin{pmatrix} \cos(k\alpha) & \sin(k\alpha) & 0 \\ -\sin(k\alpha) & \cos(k\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$w_{(i,j,k)} = s_{(i,j)} * r_k$$
$$0 \leq i \leq n; \quad 0 \leq j \leq m; \quad 0 \leq k \leq o$$