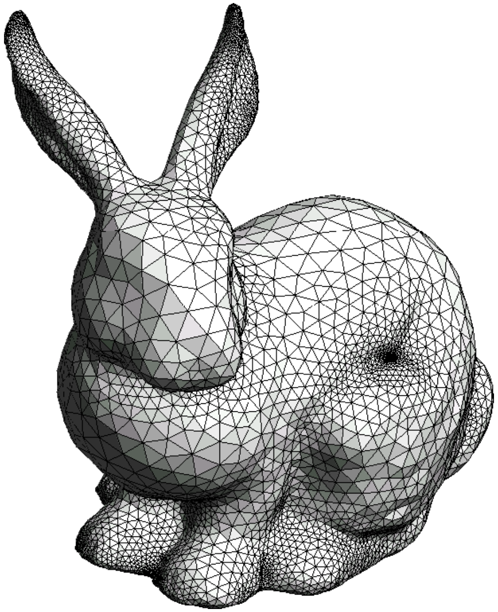

Geometric Modeling in Graphics

Part 3: Mesh simplification

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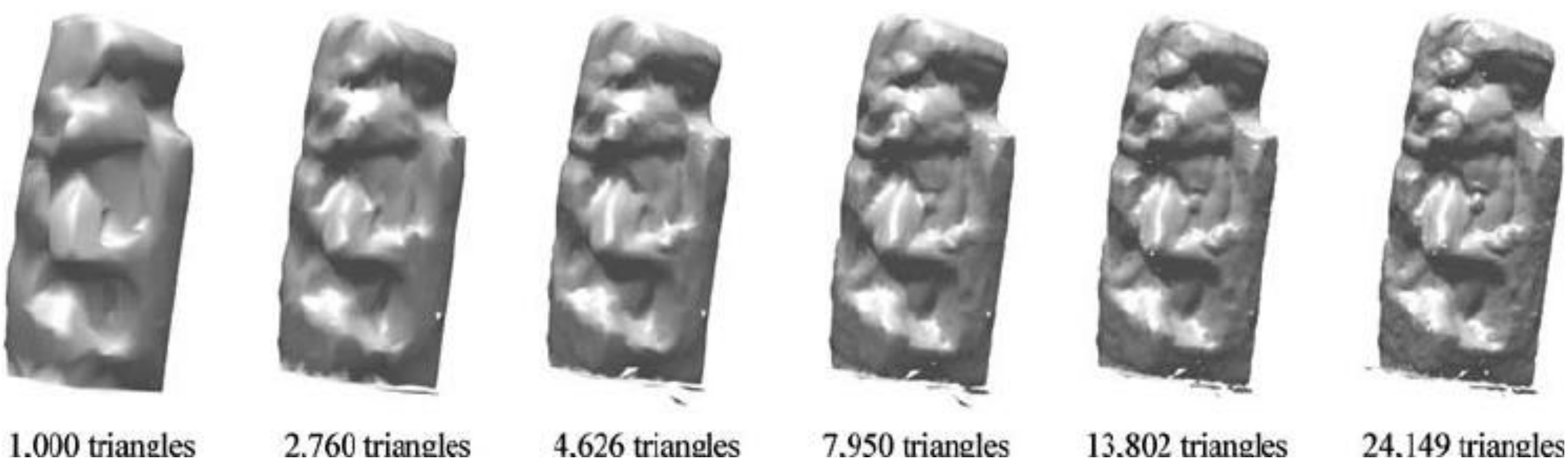
Mesh simplification

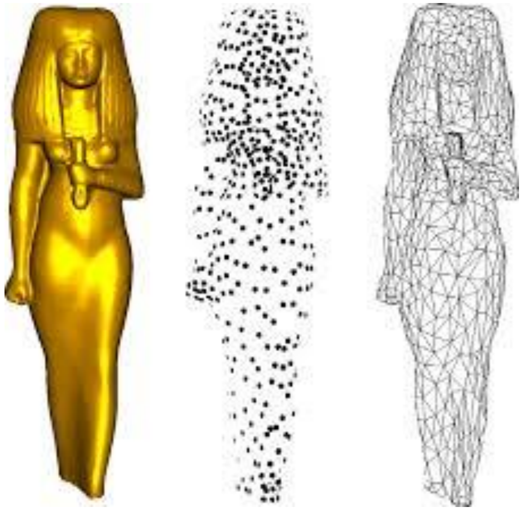
- Reducing number of vertices, edges, polygons
- Mesh decimation, mesh reduction, …
- Creating levels of detail (LOD) meshes, several versions of same mesh with different number of polygons
- Lots of algorithms, lots of similar approaches
- Comparing versions using distance (Hausdorff, …)



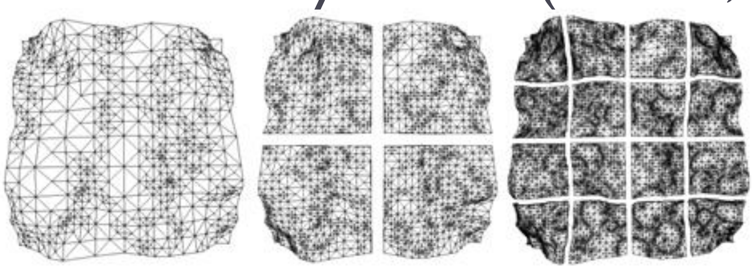
Mesh simplification

- Preserving topology (number of holes) vs reducing topology
- Static
  - Creating several levels of detail in preprocess stage
  - Almost no processing on the fly
  - Visualization-ready preparation of levels
- Dynamic
  - Level of details is created on the fly
  - Encoding continuous spectrum of details
  - Progressive transmission
- View-dependent
  - Dynamic selection of LOD based on view criteria

Simplification algorithms

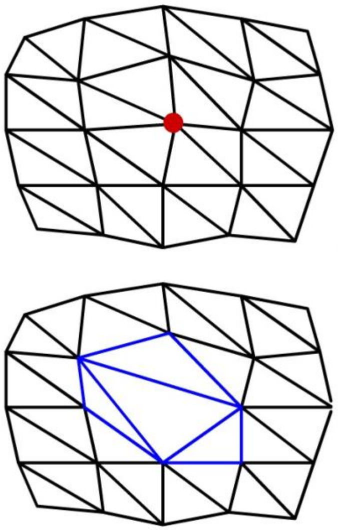


- Sampling
  - Sample mesh surface with points or voxels
  - Use smoothing on sampled points
  - Triangulate processed sampled points
  - For smooth objects
- Adaptive subdivision
  - Find base mesh as simplest level of detail
  - Levels are created from base mesh using subdivision
  - For models where base mesh is easy to find (terrain, …)

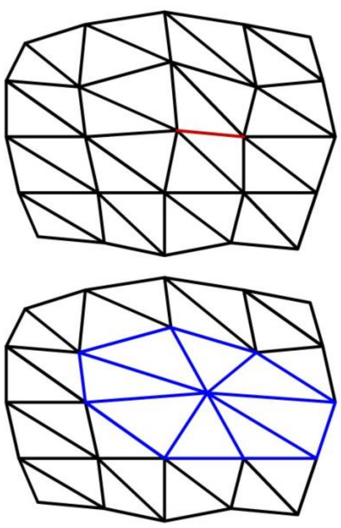


Simplification algorithms

- Decimation
  - Iteratively removing vertices or faces
  - Retriangulating hole after each step
  - Which vertex, face to remove at each step?
  - Usually simple and topology preserving
- Vertex merging
  - Iteratively collapsing two or more vertices into one
  - Which vertices to merge et each step?
  - What is new position of merged vertex?
  - Edge-collapse – merging two connected vertices
  - Can modify topology



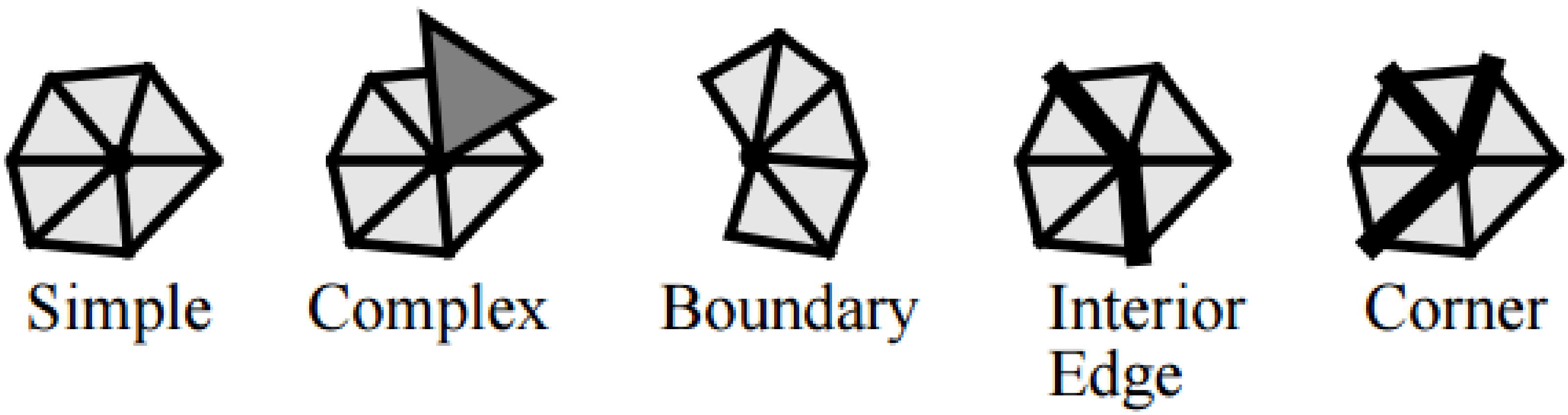
Vertex Removal



Edge Collapse

Triangle mesh decimation

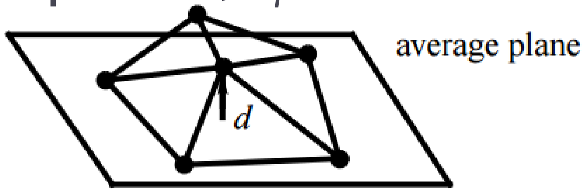
- Schroeder, Zarge, Lorenson: Decimation of Triangle Meshes
- https://webdocs.cs.ualberta.ca/~lin/ABProject/papers/4.pdf
- Deleting chosen vertex at each step of decimation and triangulating resulting hole
- 1.Characterizing local topology for each vertex
  - Feature edges defined by feature threshold angle



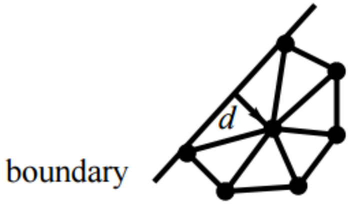
Triangle mesh decimation

- 2. Evaluating decimation criteria
  - Simple vertex **v** – distance to average plane *d*, P_i is area of triangle

$$\mathbf{N} = \frac{\sum_m \mathbf{n}_i P_i}{\sum_m P_i}, \mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|}, \quad \mathbf{x} = \frac{\sum_m \mathbf{x}_i P_i}{\sum_m P_i} \quad d = |\mathbf{n}.(\mathbf{v} - \mathbf{x})|.$$



  - Boundary and interior edge vertex – distance to line created by other two boundary vertices



  - Corner or complex vertex – usually not removed
- 3. Pick vertex with lowest criteria an remove it together with incident triangles
  - Using priority queue
  - Preserve feature edges

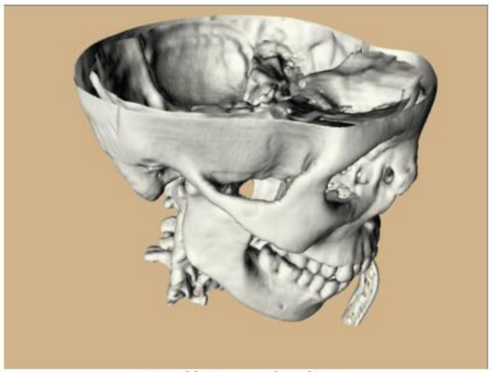
Triangle mesh decimation

- 4. Triangulate resulting hole
  - Non planar triangulation of vertices loop
  - Triangulate one (removed simple, boundary vertex) or two loops (removed interior edge vertex)
  - Generate non-intersecting, non-degenerated triangulation
  - If triangulation can not be performed, do not remove vertex and triangles
  - Use triangulation schemes based on recursive loop splitting
- 5. Finish vertex removal loop when some criterion is reached
  - Number of vertices is below threshold
  - Number of vertices is below percentage
  - Removal of any vertex will cause in non-manifold or degenerated situation

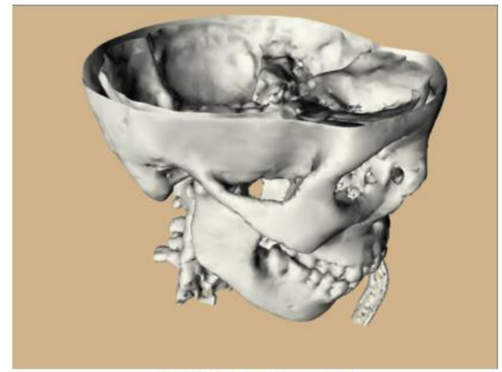
Triangle mesh decimation

Implementation

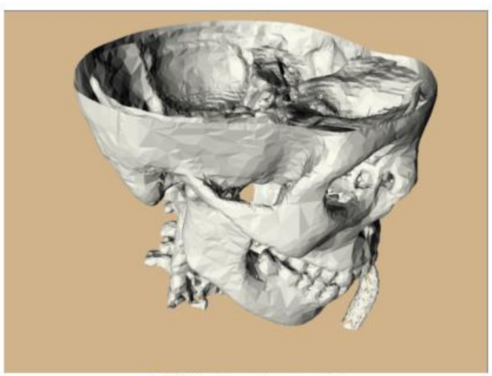
http://www.vtk.org/doc/nightly/html/classvtkDecimatePro.html



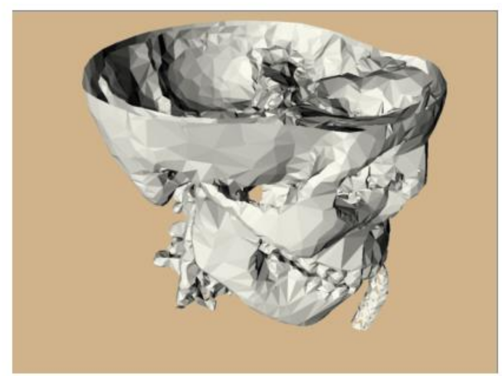
Full Resolution
(569K Gouraud shaded triangles)



75% decimated
(142K Gouraud shaded triangles)



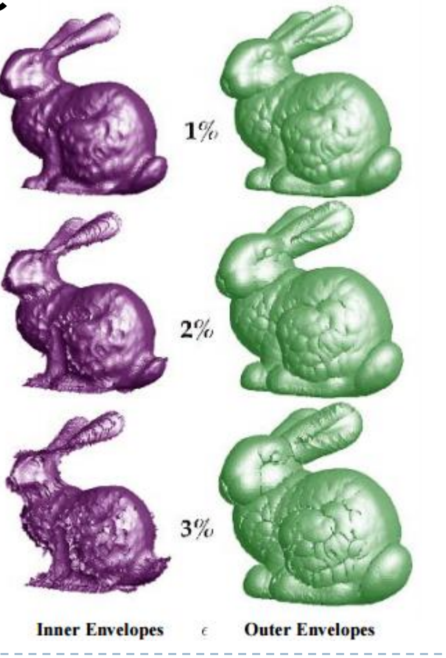
75% decimated
(142K flat shaded triangles)



90% decimated
(57K flat shaded triangles)

Simplification envelopes

- Cohen, …: Simplification Envelopes
- http://gamma.cs.unc.edu/ENVELOPES/
- Envelope – two offset surfaces, outer envelope displaces each vertex of the original mesh along its normal by ε, inner envelope displaces each vertex by −ε
- For orientable manifold triangle meshes
- Iteratively remove triangles or vertices and retriangulate the resulting holes, keeping the simplified surface within the envelopes
- Strict preservation of topology



Vertex clustering 1

- Rossignac, Borrel: Multi-Resolution 3D Approximations for Rendering Complex Scenes
- Vertex merging algorithm over uniform grid
- http://www.cc.gatech.edu/~jarek/papers/VertexClustering.pdf
- Not requiring manifold topology, not preserving topology
- 1. Assign importance for each vertex, based on sum of areas of incident triangles and „curvature“ of vertex
- 2. Triangulate faces and put 3D uniform grid over model
- 3. Merge all vertices of one cell into one with highest importance
- 4. Remove all degenerated triangles

Vertex clustering 2

- Low, Tan: Model Simplification Using Vertex Clustering
- https://www.comp.nus.edu.sg/~tants/Paper/simplify.pdf
- Floating-cell clustering, working at one vertex at a time
- Paper works in real time environment, using view-dependent LOD mesh creation
- 1.Grade each vertex, compute weight using 2 factors
  - Factor1 - Cosine of inverse of the maximum angle between all pairs of incident edges on the vertex
  - Factor 2 - Length of the longest among all of the edges incident upon the vertex
  - Sort vertices based on weight
- 2. Triangulate each face

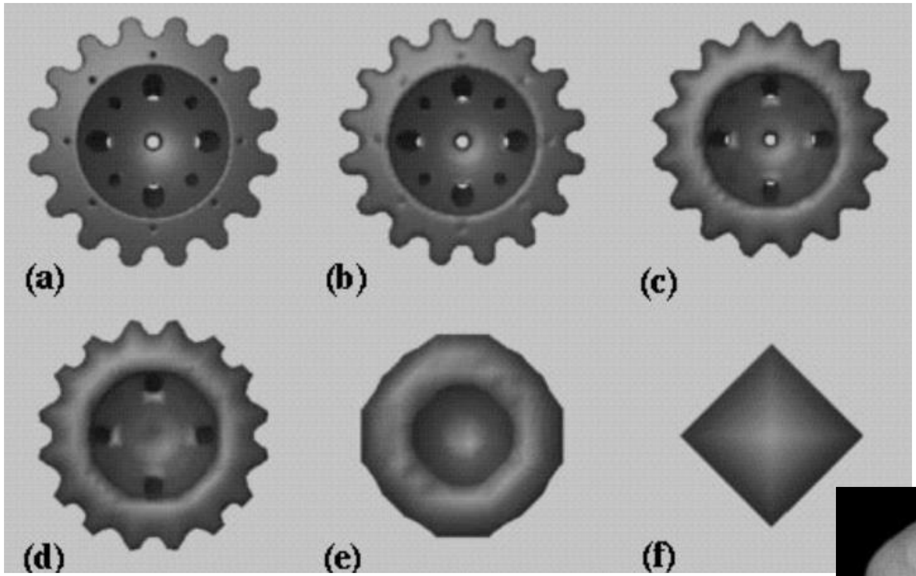
Vertex clustering 2

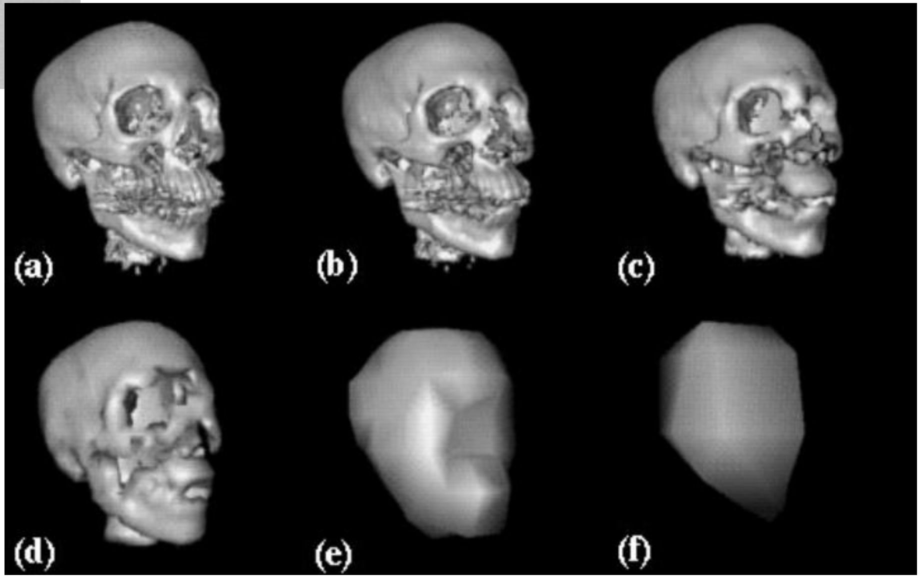
- 3. Put box with user defined size at vertex with highest weight
  - Vertex is at the center of box
- 4. Merge all vertices that are inside box to one vertex with highest weight
  - Remove merged vertices from list
  - Remove degenerated triangles
- 5. Repeat merging process for next highest weighted vertex
- 6. Repeat process until some threshold is reached
- Worse control over number of vertices in simplified mesh
- Because of sorting, time complexity is O(nlog(n))

Voxel-based simplification

- He, Hong, …:Voxel Based Object Simplification
- https://www.cs.umd.edu/gvil/papers/he_voxel.pdf
- Requiring well-defined, closed-mesh, manifold mesh
- Superimposing a 3D uniform grid of voxels over the polygonal geometry
- Sampling mesh by assigning each voxel a value of 0 or 1 according to whether the sample point of that voxel lies inside or outside the object
- Applying low-pass filter on voxel values – Gauss, …
- Using Marching cubes to generate polygonal mesh from filtered values in uniform grid using isovalue 0.5
- Good for meshes without very sharp vertices or edges

Voxel-based simplification





QEM simplification

- Garland, Heckbert: Surface Simplification Using Quadric Error Metrics
- http://cseweb.ucsd.edu/~ravir/190/2016/garland97.pdf
- Iterative contraction of vertex pairs, possible connection of two unconnected vertices that are close enough
- New computation of vertex-merge error
- For triangular meshes
- No requirement for manifold topology
- No topology preservation
- Probably best combination of efficiency, fidelity, and generality

QEM pair selection

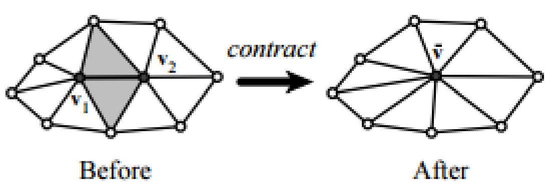
- Picking valid vertex pair ($\mathbf{v}_1,\mathbf{v}_2$) for contraction at initialization time
  - $\mathbf{v}_1,\mathbf{v}_2$ is an edge
  - $|\mathbf{v}_1-\mathbf{v}_2|<T$, T is user defined threshold
- Threshold T=0 gives simple edge contracting algorithm
- Positive T gives algorithm ability to connect unconnected parts and to change genus of mesh
- With bigger T, we can move to $O(n^2)$ pairs, slowing algorithm significantly
- When pair ($\mathbf{v}_1,\mathbf{v}_2$) is contracted into vertex $\mathbf{v}$, all candidate pairs containing $\mathbf{v}_1$ and $\mathbf{v}_2$ are updated with $\mathbf{v}$

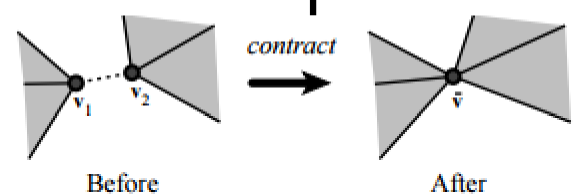
QEM error approximation

- Introducing cost of contraction to select one best pair to contract during a given iteration
- Associating 4x4 matrix $\mathbf{Q}_\mathbf{v}$ with each vertex $\mathbf{v}$
  - Construct plane p: $ax+by+cz+d = 0, a^2+b^2+c^2=1$ for each triangle incident to vertex $\mathbf{v}$
  - Compute fundamental error quadric matrix $\mathbf{K}_\mathbf{p}$ $\quad \mathbf{K}_\mathbf{p} = \mathbf{p}\mathbf{p}^T = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$
  - $\mathbf{Q}_\mathbf{v}$ is then sum of all $\mathbf{K}_\mathbf{p}$
- Error (cost) at vertex v is $\Delta(\mathbf{v})=\mathbf{v}^T\mathbf{Q}_\mathbf{v}\mathbf{v}$
- After contraction $(\mathbf{v}_1,\mathbf{v}_2)\rightarrow \mathbf{v}$, the new error matrix is $\mathbf{Q}_\mathbf{v}=\mathbf{Q}_{\mathbf{v}_1}+\mathbf{Q}_{\mathbf{v}_2}$
- After contraction $(\mathbf{v}_1,\mathbf{v}_2)\rightarrow \mathbf{v}$, the position of v is such that it minimizes $\Delta(\mathbf{v})$, e.g. $\mathbf{v}=\mathbf{Q}_\mathbf{v}^{-1}\mathbf{0}^T$
- Cost of contraction $(\mathbf{v}_1,\mathbf{v}_2)\rightarrow \mathbf{v}$ is $\Delta(\mathbf{v})=\mathbf{v}^T\mathbf{Q}_\mathbf{v}\mathbf{v}$

QEM sum

- 1. Compute the Q matrices for all the initial vertices.
- 2. Select all valid pairs.
- 3. Compute the optimal contraction target v for each valid pair $(\mathbf{v_1}, \mathbf{v_2})$. The error $\mathbf{v}^T(\mathbf{Q}_{\mathbf{v}_1}+\mathbf{Q}_{\mathbf{v}_2})\mathbf{v}$ of this target vertex becomes the cost of contracting that pair.
- 4. Place all the pairs in a heap keyed on cost with the minimum cost pair at the top.
- 5. Iteratively remove the pair $(\mathbf{v_1}, \mathbf{v_2})$ of least cost from the heap, contract this pair, and update the costs of all valid pairs involving $\mathbf{v_1}, \mathbf{v_2}$. Remove also all collapsed triangles.





QEM

Implementation

http://www.cs.cmu.edu/~./garland/quadrics/qslim.html

Figure 14: **Original.** Bones of a human's left foot (4,204 faces). Note the many separate bone segments.

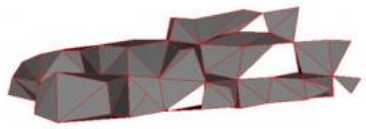


Figure 15: **Uniform Vertex Clustering.** 262 face approximation ($11 \times 4 \times 4$ grid). Indiscriminate joining destroys approximation quality.

Figure 16: **Edge Contractions.** 250 face approximation. Bone segments at the ends of the toes have disappeared; the toes appear to be receding back into the foot.

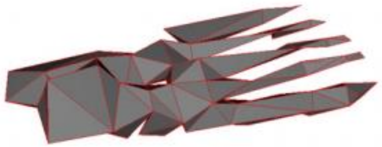
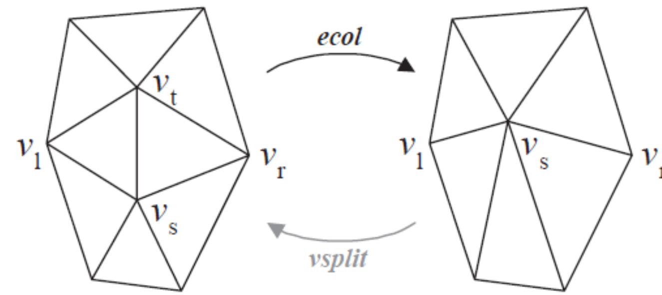


Figure 17: **Pair Contractions.** 250 face approximation ($t = 0.318$). Toes are being merged into larger solid components. No receding artifacts. This model now contains 61 non-manifold edges.

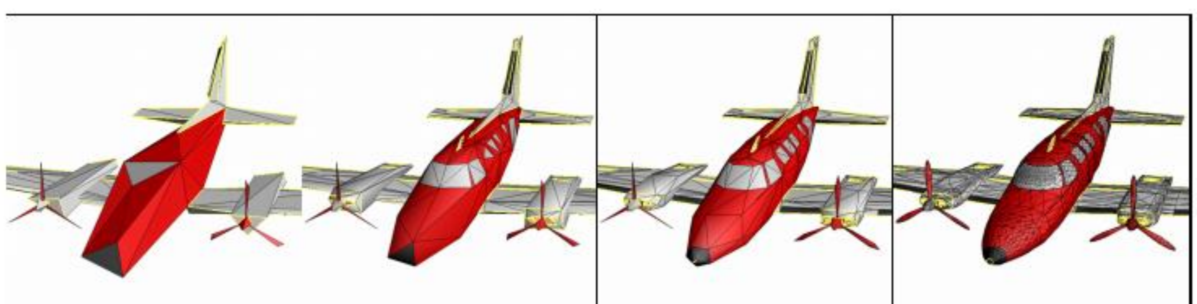
Progressive meshes



- Hoppe
- http://research.microsoft.com/en-us/um/people/hoppe/proj/pm/
- Edge-collapse simplification scheme on triangular meshes
- Requiring manifold topology, preserving topology
- Introducing energy function for mesh
- Algorithm evaluates all edges that can be collapsed according to their effect on energy function and sorts them into a priority queue
- After collapse of edge with lowest energy, energy reevaluates and resorts nearby edges into the queue

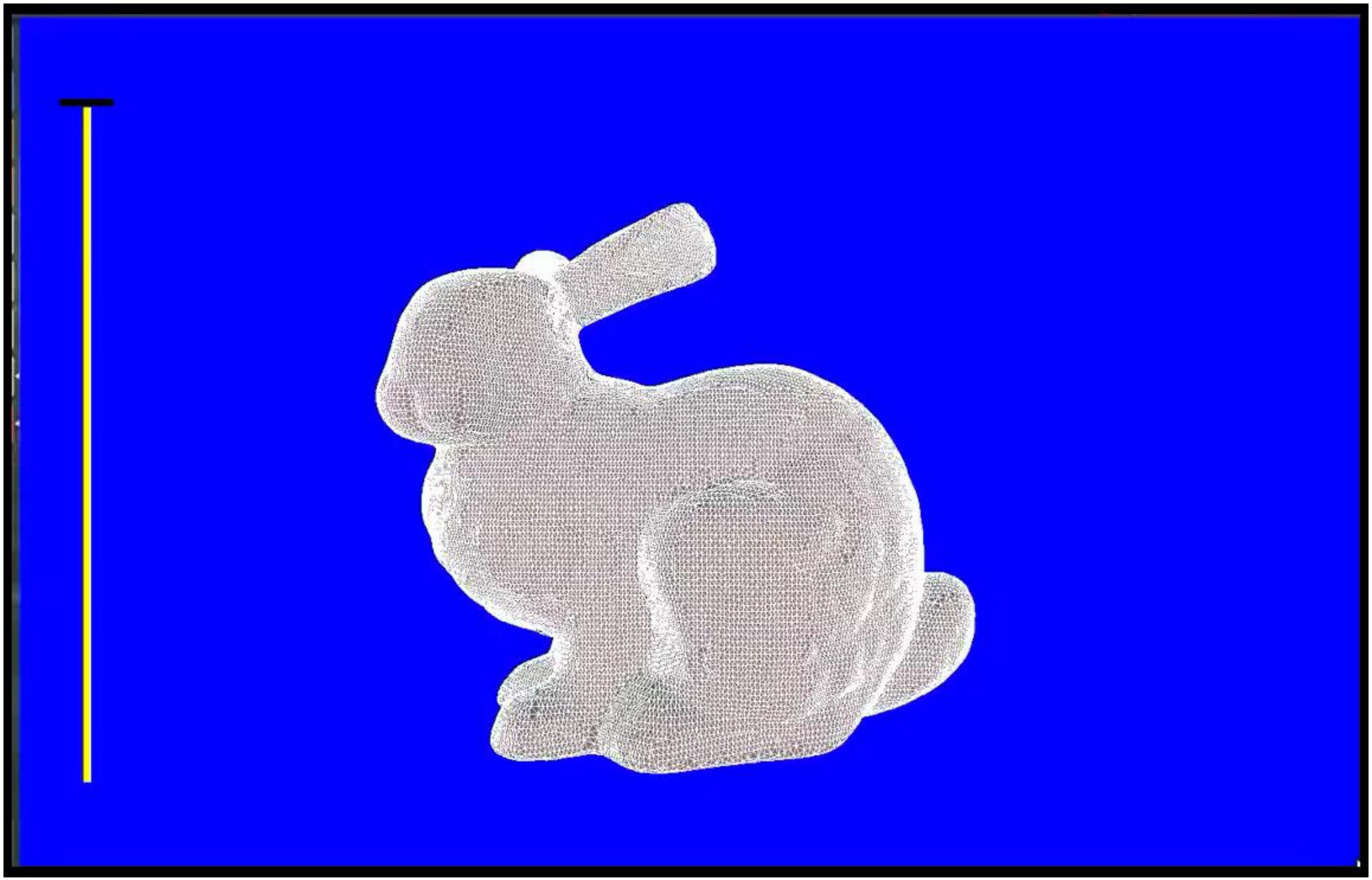
Progressive meshes

- Position of new vertex vs after (v_s,v_t) collapse can be v_s,v_t or $(0.5v_s+0.5v_t)$
- Process repeats until topological constraints prevent further simplification – base mesh
- Vertex split(vsplit) – inverse to edge collapse (ecol)
- Progressive mesh – from base mesh to any level of details using several consecutive vsplits
- Using in mesh compression, progressive transmission



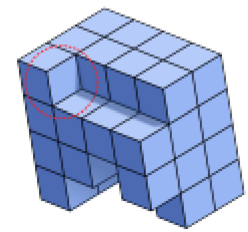
(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces) (c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M}=M^n$ (13,546 faces)

Progressive meshes



Quad mesh simplification

- Tarini, …: Practical quad mesh simplification
- http://vcg.isti.cnr.it/Publications/2010/TPCPP10/
- Requiring regularity over quad mesh
- Composing all iterations of quad mesh simplification to be as regular (as *homeometry*) as possible
  - Edges has same length *l*
  - Diagonals of faces has same length *l.sqrt(2)*



- How far is mesh *M* from being homeometric – objective function
  - e spans over all edges of mesh
  - *d* spans over all edges of mesh

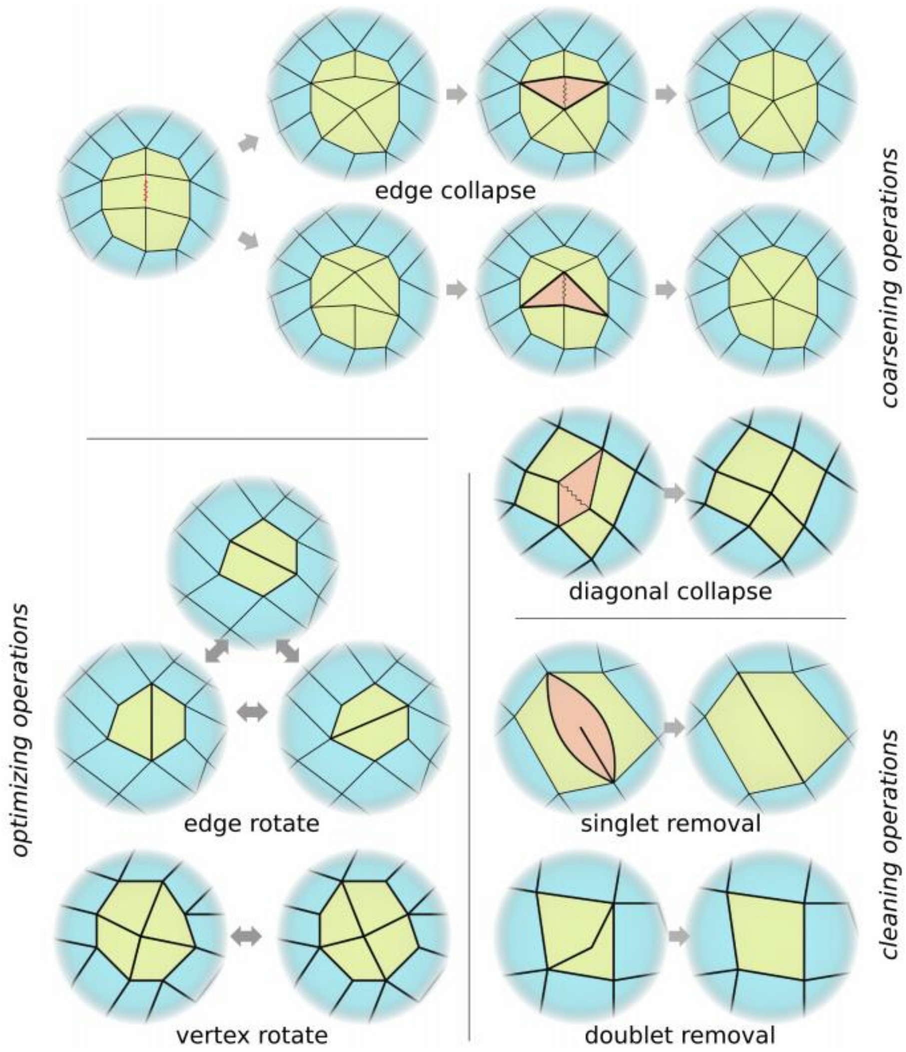
$$\mu = \sqrt{Area(M)/|M|},$$

$$\sum_{e \in M^E} (|e| - \mu)^2 + \sum_{d \in M^D} (|d| - \sqrt{2}\mu)^2$$

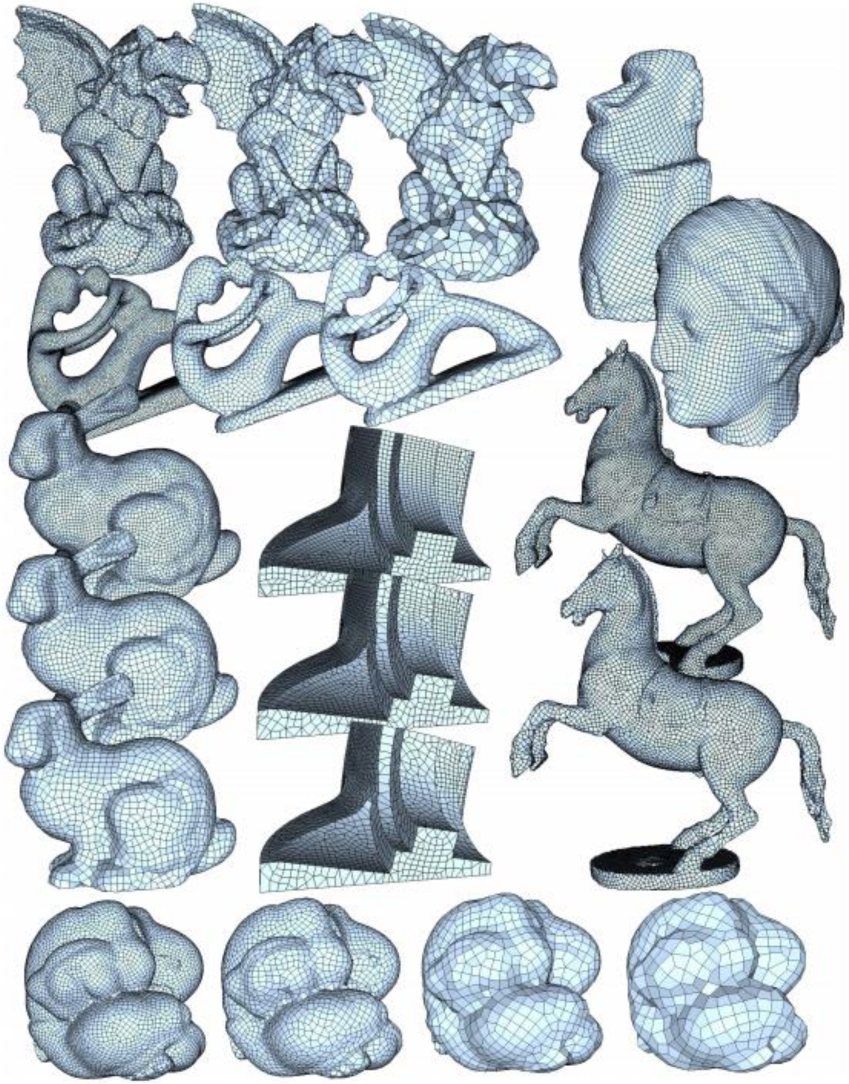
Quad mesh simplification

- 0. [Convert input mesh into quad mesh M_0]
- 1. Initial global smoothing of mesh M_0 (minimizing function)
- 2. Iteratively process mesh M_i to produce mesh M_{i+1} until user-defined criterion is met. In each loop:
  - a. for a fixed number of times:
    - i. choose shortest edge or diagonal
    - ii. perform any profitable local optimizing-operation, until none is available
    - iii. select and perform a local coarsening-operation and cleaning operation on chosen elements such that operations minimizes objective function as best as possible
  - b. local smoothing
- 3. Final global smoothing of mesh M_n

Quad mesh local operations

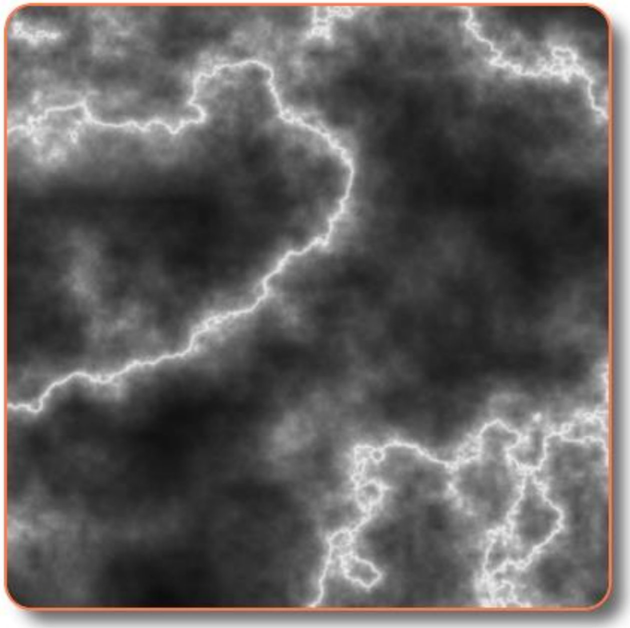


Quad mesh simplification



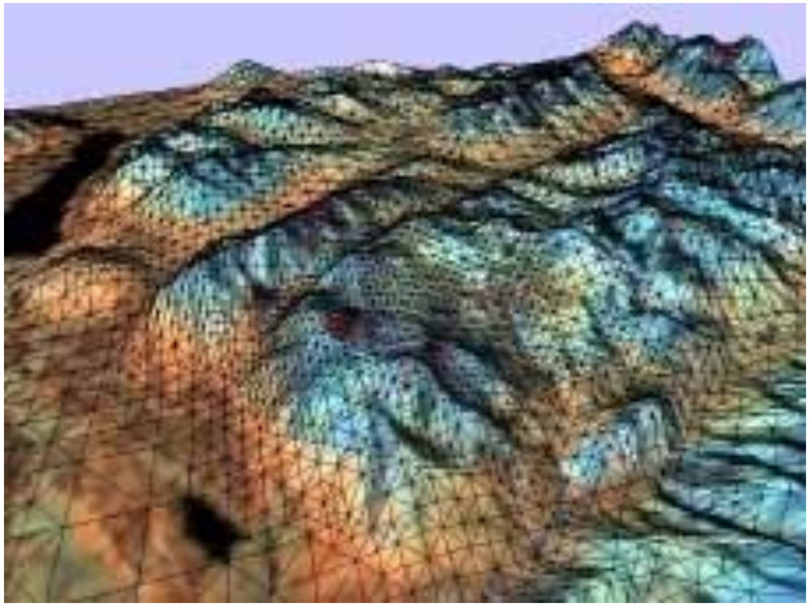
Terrain visualization

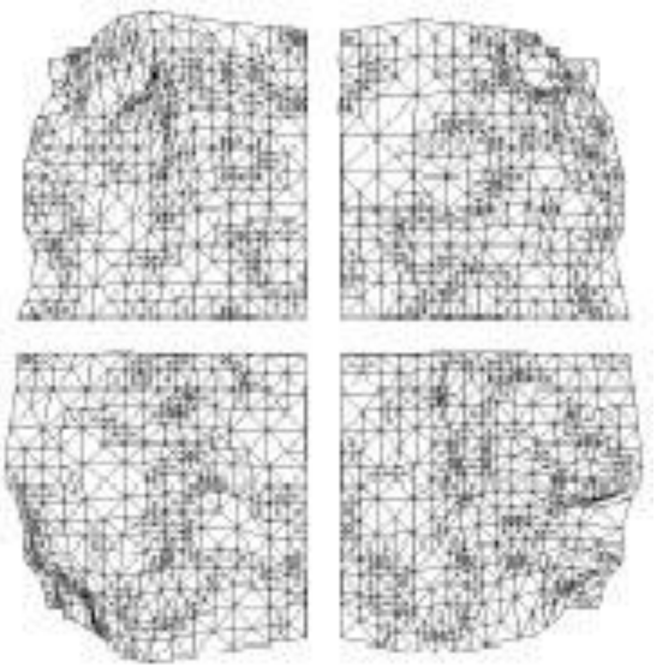
- View above terrain surface – some parts are close, some away – using LOD (Level Of Detail), each part of terrain is rendered in some detail based on distance from camera
- Needed structure for storing all levels of detail for each part of terrain
- Terrain
  - Height field



Terrain visualization

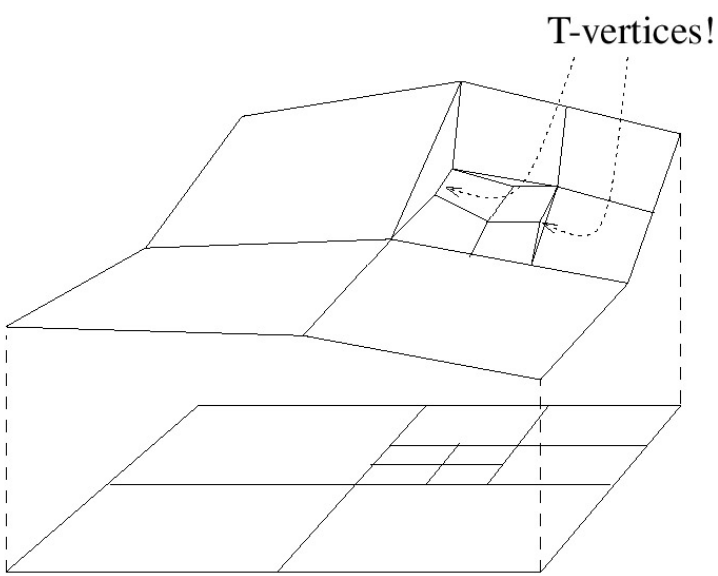
- Creating complete quadtree over height filed
- Traversing tree during rendering – based on distance of camera and node area, the traverse is stopped or continued
- View-dependent, subdivision based, dynamic LOD creation

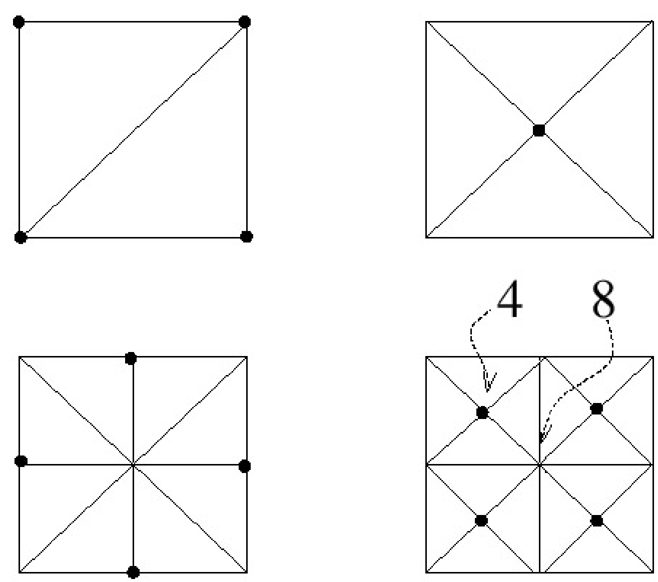


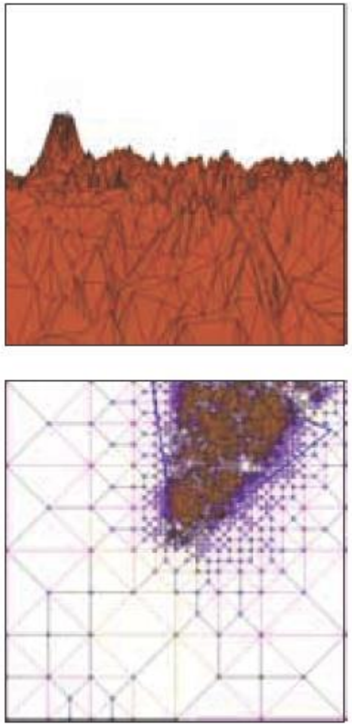


Terrain visualization

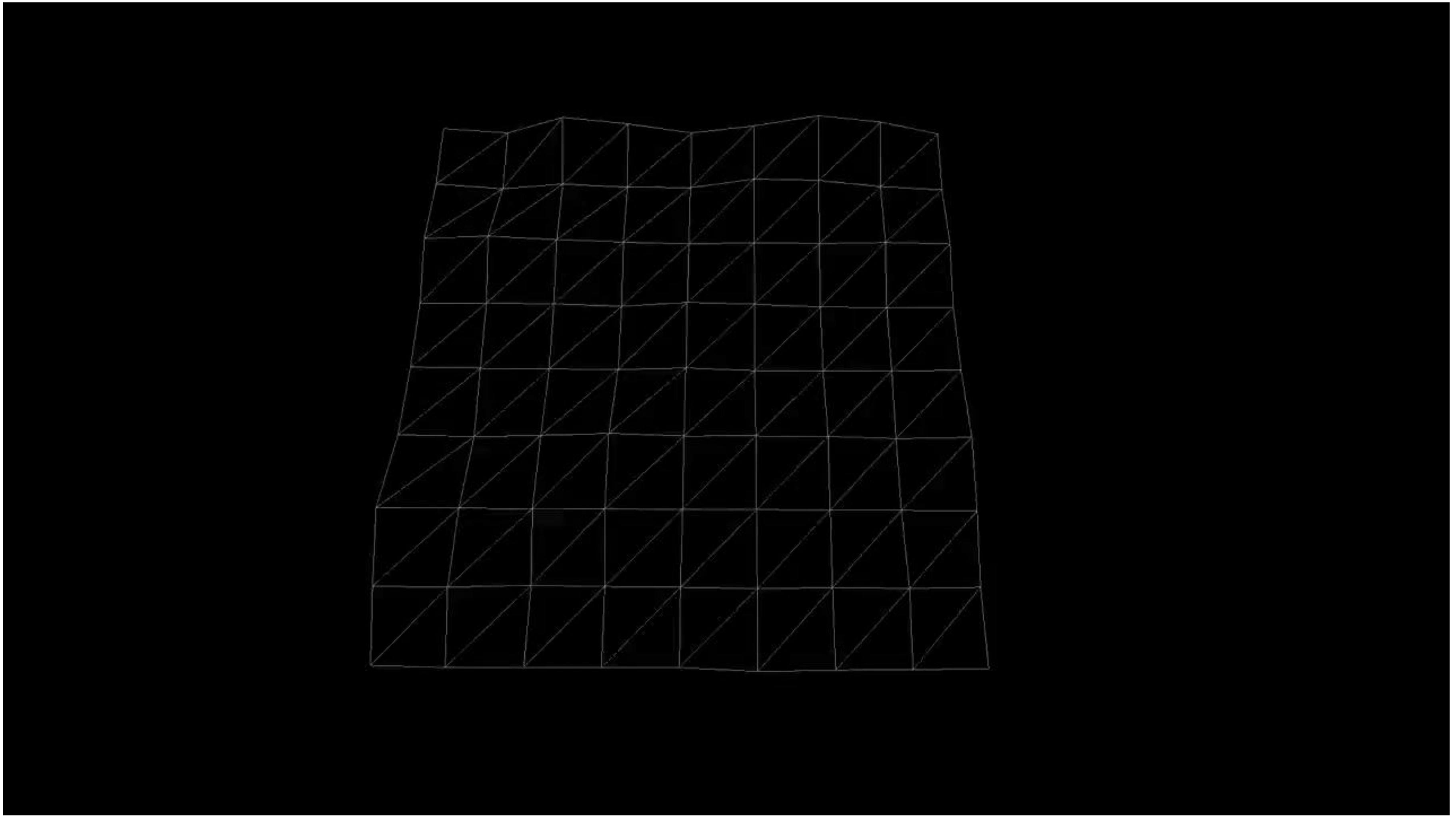
- Problems with the edge between areas on different levels in quadtree
- Solution using triangulation= connection of two consecutive quadtrees

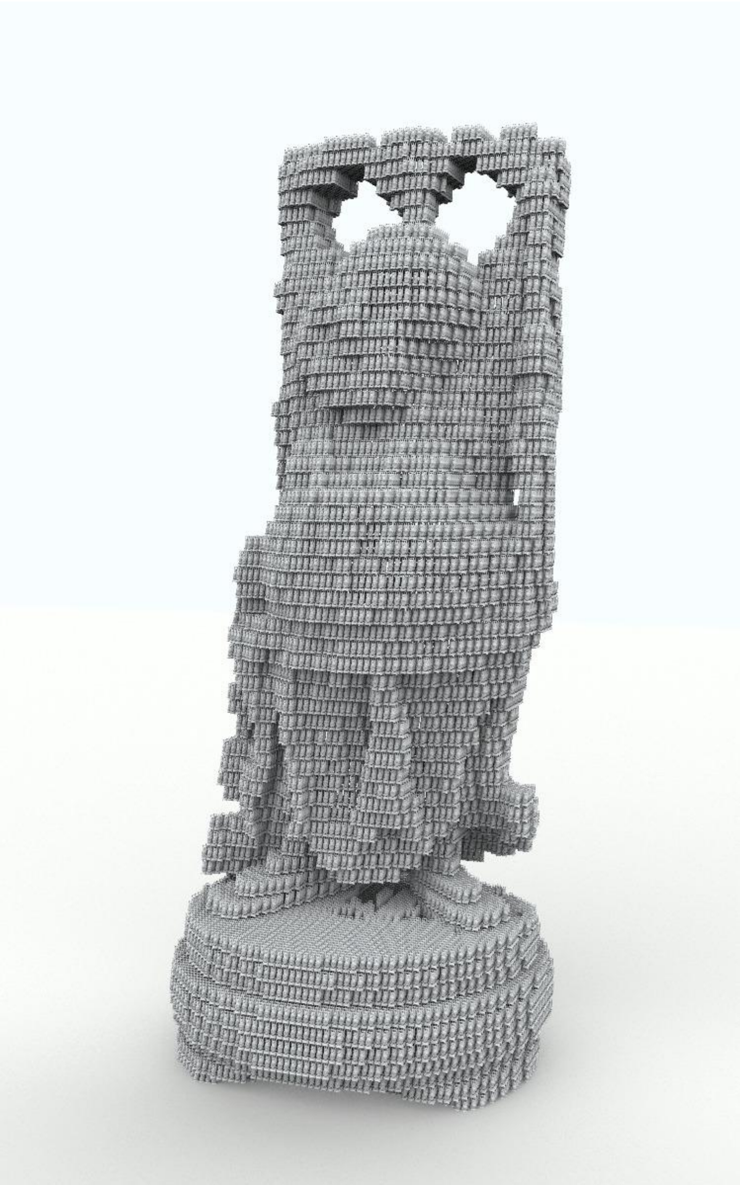






Terrain visualization





The End
for today